Calculus AB

Slope Fields

Draw the slope field for the following differential. Then draw the graph of the solution through the given point. Then solve the differential equation for the given point.

1) $dy = x^3\ dx$ (0, -2)
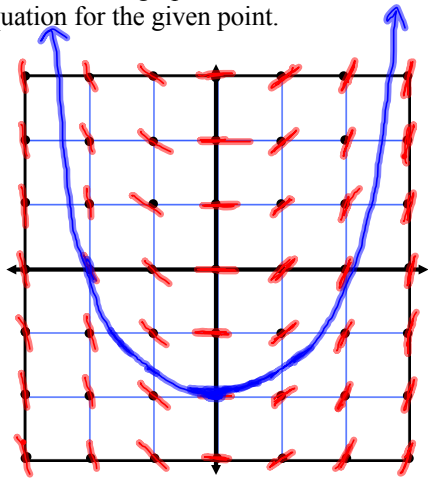
(x, m)

$(0, 0)$
$(1, 1)$
$(2, 8)$
$(3, 27)$
$(-1, -1)$
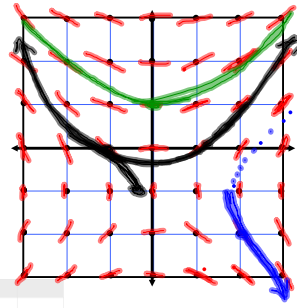$(-2, -8)$
$(-3, -27)$

$\frac{1}{4}x^4 - 2$

Draw the slope field for the following differential. Draw the graph of the solution through the given point. Then solve the differential equation for each point.

2) $dy = \dfrac{x}{y+1}\,dx$

(x, y, m)
$(0, 0, 0)$

a) $(-1, 0)$ $\langle 1, 0, 1 \rangle$
b) $(0, 1)$ $(2, 0, 2)$
c) $(2, -2)$ $(3, 0, 3)$

$(0, 1, 0)$ $(0, 2, 0)$ $(-1, 0, -1)$
$(1, 1, \tfrac{1}{2})$ $(1, 2, \tfrac{1}{3})$ $(-2, 0, -2)$
$(2, 1, 1)$ $(-3, 0, -3)$
$(3, 1, \tfrac{3}{2})$
$(-1, 1, -\tfrac{1}{2})$
$(-2, 1, -1)$
$(-3, 1, -\tfrac{3}{2})$

Solved on next page.

x	y	m
-3	0	-3
-2	0	-2
-1	0	-1
0	0	0
1	0	1
2	0	2
3	0	3

x	y	m		x	y	
-3	1	-1.5		-3	-1	
-2	1	-1		-2	-1	
-1	1	-0.5		-1	-1	
0	1	0		0	-1	
1	1	0.5		1	-1	
2	1	1		2	-1	
3	1	1.5		3	-1	

x	y	m		x	y	m
-3	2	-1		-3	-2	3
-2	2	-0.67		-2	-2	2
-1	2	-0.33		-1	-2	1
0	2	0		0	-2	0
1	2	0.333		1	-2	-1
2	2	0.667		2	-2	-2
3	2	1		3	-2	-3

x	y	m		x	y	m
-3	3	-0.75		-3	-3	1.5
-2	3	-0.5		-2	-3	1
-1	3	-0.25		-1	-3	0.5
0	3	0		0	-3	0
1	3	0.25		1	-3	-0.5
2	3	0.5		2	-3	-1
3	3	0.75		3	-3	-1.5

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2) $dy = \dfrac{x}{y+1}\,dx$

$\displaystyle \int (y+1)\,dy = \int x\,dx$ (separation of variables)

$\tfrac{1}{2}y^2 + y = \tfrac{1}{2}x^2 + C$

$y^2 + 2y = x^2 + C$

$y^2 + 2y + 4 = x^2 + C$

$\boxed{-2 \pm \sqrt{x^2 + C} = y}$

$(y+2)^2 = x^2 + C$

$|y + 2| = \sqrt{x^2 + C}$